

Characterization of coherent impurity effects in solid state qubits

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We propose a characterization of the effects of bistable *coherent impurities* in solid state qubits. We introduce an effective impurity description in terms of a tunable spin-boson environment and solve the dynamics for the qubit coherences. The dominant rate characterizing the asymptotic time limit is identified and signatures of non-Gaussian behavior of the quantum impurity at intermediate times are pointed out. An alternative perspective considering the qubit as a measurement device for the spin-boson impurity is proposed.

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Coherent nanodevices are inevitably exposed to fluctuations due to the solid-state environment. Well studied examples are charged impurities and stray flux tubes which are sources of telegraphic noise in a wide class of metallic devices. Large amplitude low-frequency (mostly $1/f$) noise, ubiquitous in amorphous materials [1], is also routinely measured in single-electron-tunneling devices [2]. Noise sources are sets of impurities located in the oxides and in the substrate, each producing a bistable stray polarization. Telegraphic noise has also been observed in semiconductor and superconductor based nanocircuits [3]. The possible presence of impurities entangled with the device has been suggested in [4]. Recent experiments on Josephson qubits indicated that charged impurities may also be responsible for noise [5] exhibiting an ohmic power spectrum at GHz-frequencies. Different theoretical models have been proposed aiming to a unified description of broadband noise sources. They share the common idea that the variety of observed features are due to the dynamics of ensembles of bistable impurities [5, 6, 7, 8]. In particular in Ref. [8] it has been proposed that a noise power spectrum compatible with the observed relaxation of charge-Josephson qubits [5] can be obtained if sets of *coherent* impurities are considered.

Solid-state noise also determines dephasing. This issue has attracted a great deal of interest in recent years since it has been recognized as a severe hindrance for the implementation of quantum hardware in the solid state. The effect of slow noise due to ensembles of thermal [9, 10] and non-thermal [7] fluctuators has been addressed. Slow noise explains the non-exponential suppression of coherent oscillations observed when repeated measurements are performed [11, 12]. In addition fluctuations active *during time evolution* represent an unavoidable limitation even when a single-shot measurement scheme or dynamical decoupling protocols [13] are available. Note that at experimental temperatures (~ 10 mK) quantum impurities may have a significant influence.

In this Communication we investigate qubit dephasing during time evolution due to coupling to a coherent impurity. The *full* qubit dynamics is solved in the regime where qubit relaxation processes are absent. We show how the coherent and non-linear dynamics of the impurity is reflected in the qubit behavior. We identify regimes characterized by strong qubit - impurity back-action. Specifically, we discuss dependence on the impurity preparation and beating phenomena. An alternative interpretation with the qubit acting as a measurement device for the impurity is presented at the end of this Communication.

Model.— We model the impurity as a two-state system, $\mathcal{H}_I = -\frac{1}{2}\varepsilon\tau_z - \frac{1}{2}\Delta\tau_x$, coupled to the qubit (σ) via $\mathcal{H}_{QI} = -\frac{1}{2}v\sigma_z\tau_z$ ($\hbar = 1$). This anisotropic coupling has been discussed for charge qubits, where it models the electrostatic interaction [8, 9, 10]. In this case the two physical states ($\tau_z \rightarrow \pm 1$) correspond to a bistable stray polarization of the qubit. They are viewed as the ground states of a double-well deformation potential, the impurity oscillating coherently between them with frequency $\Omega_I = \sqrt{\varepsilon^2 + \Delta^2}$. Dissipative transitions between the minima come from the interaction with a bosonic bath [14] ($\mathcal{H}_B = \sum_{\alpha} \omega_{\alpha} a_{\alpha}^{\dagger} a_{\alpha}$) via $\mathcal{H}_{IB} = -\frac{1}{2}\hat{X}\tau_z$. The operator $\hat{X} = \sum_{\alpha} \lambda_{\alpha}(a_{\alpha} + a_{\alpha}^{\dagger})$ is a collective displacement with ohmic power spectrum $S(\omega) = 2\pi K\omega \coth \frac{\omega}{2T}$ with a high-energy cutoff at ω_c ($k_B = 1$) [14]. This *spin-boson environment* (SBE) may induce a variety of qubit dynamical behaviors, since its degree of coherence depends on K and on temperature T [14]. For instance for weak damping, $K \ll 1$ a crossover occurs between a low “impurity temperature”, $T \ll \Omega_I$ regime, where the impurity performs damped oscillations, to the regime of incoherent dynamics if $T \gg \Omega_I$ (white noise $S(\omega) \approx 4\pi KT$) [15].

We assume that the qubit Hamiltonian conserves σ_z , therefore the impurity induces *pure dephasing* [14] with no relaxation of the qubit [16]. This regime is very interesting since energy exchange processes do not blur deco-

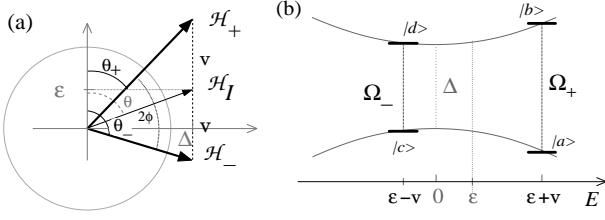


FIG. 1: (a) Impurity Bloch sphere. An isolated impurity \mathcal{H}_I defines the mixing angle $\theta = \arctan \Delta/\varepsilon$, \mathcal{H}_{\pm} define $\theta_{\pm} = \arctan \Delta/(\varepsilon \pm v)$. (b) Impurity bands $\pm\sqrt{E^2 + \Delta^2}$: impurity energy splittings depend on the qubit state, $\Omega_{\pm} = \sqrt{(\varepsilon \pm v)^2 + \Delta^2}$. Eigenstates of \mathcal{H}_0 , are $\{|i\rangle\}$, $i = a, b, c, d$. Conservation of σ_z allows only intra-doublet processes, $a \leftrightarrow b$, $c \leftrightarrow d$.

herence of the qubit, which is then maximally sensitive to the SBE dynamics. Pure dephasing due to Fano impurities was addressed in [9], recently the asymptotic dynamics has been studied [17]. This model corresponds to a over-damped impurity (SBE at $K = \frac{1}{2}$), here we consider $K \ll 1$ where the impurity may behave coherently.

Method and analytic results.— For pure dephasing the qubit Hamiltonian can be gauged away by a proper rotation. In this picture we consider the reduced density matrix $\rho(t)$ describing the entangled qubit-impurity system. For $K \ll 1$ the interaction with the bosonic bath is studied by the Born-Markov master equation (ME) [18]

$$\partial_t \rho(t) = -i[\mathcal{H}_0, \rho(t)] - \int_0^\infty dt' \left\{ \frac{1}{4} S(t') [\tau_z, [\tau_z(t'), \rho(t)]] + \frac{i}{2} \chi(t') [\tau_z, [\tau_z(t'), \rho(t)]_+] \right\}, \quad (1)$$

where $\mathcal{H}_0 = \mathcal{H}_{QI} + \mathcal{H}_I$ is the undamped Hamiltonian. Here, the transform $S(t)$ of the power spectrum and the bath susceptibility $\chi(t) = -i\langle [\dot{X}(t), \dot{X}(0)]_+ \rangle \Theta(t)$ enter the damping term. We introduce the *conditional* Hamiltonians of the impurity $\mathcal{H}_{\pm} = -\frac{1}{2}(\varepsilon \pm v) \tau_z - \frac{1}{2} \Delta \tau_x$, (see Fig. 1) and the eigenvectors of \mathcal{H}_0 , $\{|i\rangle\}$, which are factorized in eigenstates of σ_z and of \mathcal{H}_{\pm} [15]. The qubit dynamics at pure dephasing is described by the coherences $\langle \sigma_{\pm}(t) \rangle = \text{Tr}[\rho(t) (\sigma_x \pm i\sigma_y) \otimes \mathbb{1}_{\tau}]$, and in particular

$$\langle \sigma_{-}(t) \rangle = [\rho_{ac}(t) + \rho_{bd}(t)] \cos \phi + [\rho_{ad}(t) - \rho_{bc}(t)] \sin \phi,$$

where $\phi = \frac{1}{2}(\theta_{-} - \theta_{+})$ is a combination of the mixing angles of \mathcal{H}_{\pm} (Fig. 1). Since σ_z is conserved, the damping tensor presents only four non vanishing 4×4 diagonal blocks. We focus on the block acting on the terms entering $\langle \sigma_{-}(t) \rangle$. Performing a partial secular approximation within this block, we get two sets of decoupled equations for ρ_{ac}, ρ_{bd} and ρ_{ad}, ρ_{bc} . We quote here the first set

$$\begin{pmatrix} \dot{\rho}_{ac}(t) \\ \dot{\rho}_{bd}(t) \end{pmatrix} = \begin{pmatrix} i\delta - \Gamma_1 & \Gamma_{12} \\ \Gamma_{21} & -i\delta - \Gamma_2 \end{pmatrix} \begin{pmatrix} \rho_{ac}(t) \\ \rho_{bd}(t) \end{pmatrix}, \quad (2)$$

where $\delta = \frac{1}{2}(\Omega_{+} - \Omega_{-})$, Fig. 1. The rates Γ_i , describing dissipative transitions and pure dephasing processes

between the 4-states and the bosonic bath, read

$$\begin{aligned} \Gamma_{1,2} &= \alpha_{+}^2 \Gamma_{\mp}(\Omega_{+}) + \alpha_{-}^2 \Gamma_{\mp}(\Omega_{-}) + \eta_s S(0), \\ \Gamma_{12,21} &= \alpha_{+} \alpha_{-} [\Gamma_{\pm}(\Omega_{+}) + \Gamma_{\pm}(\Omega_{-})], \\ \alpha_{\pm} &= \frac{1}{2\sqrt{2}} \sin \theta_{\pm}; \quad \eta_s = \frac{1}{2} \sin^2 \bar{\theta} \sin^2 \phi, \end{aligned} \quad (3)$$

where $\bar{\theta} = \frac{1}{2}(\theta_{+} + \theta_{-})$. Here $\Gamma_{\pm}(\omega) = 2\pi K\omega [\coth(\frac{\omega}{2T}) \pm 1]$, are the impurity emission (+) and absorption (−) rates of energy ω . The elements ρ_{ad}, ρ_{bc} satisfy similar equations with δ replaced by $\Omega = \frac{1}{2}(\Omega_{+} + \Omega_{-})$ and rates

$$\begin{aligned} \Gamma_{3,4} &= \alpha_{+}^2 \Gamma_{\mp}(\Omega_{+}) + \alpha_{-}^2 \Gamma_{\pm}(\Omega_{-}) + \eta_c S(0), \\ \Gamma_{34,43} &= \alpha_{+} \alpha_{-} [\Gamma_{\pm}(\Omega_{+}) + \Gamma_{\mp}(\Omega_{-})], \\ \eta_c &= \frac{1}{2} \cos^2 \bar{\theta} \cos^2 \phi. \end{aligned}$$

Diagonalization of Eq. (2) and of the corresponding set for ρ_{ad}, ρ_{bc} yields the eigenvalues

$$\begin{aligned} \lambda_{1,2} &= -\frac{\Gamma_1 + \Gamma_2}{2} \pm \frac{1}{2} \sqrt{(2i\delta + \Gamma_2 - \Gamma_1)^2 + 4\Gamma_{12}\Gamma_{21}}, \\ \lambda_{3,4} &= -\frac{\Gamma_3 + \Gamma_4}{2} \pm \frac{1}{2} \sqrt{(2i\Omega + \Gamma_4 - \Gamma_3)^2 + 4\Gamma_{34}\Gamma_{43}}. \end{aligned} \quad (4)$$

The explicit form of $\langle \sigma_{-}(t) \rangle$ depends on the initial conditions for $\rho(t)$. Because of the high accuracy of preparation presently achieved in solid state implementations, factorized qubit-impurity states $\rho(0) = \rho_{\sigma}(0) \otimes \rho_{\tau}(0)$, represent a realistic scenario. The impurity initial state is instead out of the experimentalist control, thus we choose $\rho_{\tau}(0) = \frac{1}{2}(\mathbb{1}_{\tau} + p_z \tau_z)$, p_z being the initial average of τ_z . The impurity starts from a totally unpolarized state for $p_z = 0$, from a pure state if $p_z = \pm 1$. This class of initial states guarantees the positivity of the dynamical process ensuing from Eq.(1). With this choice we find

$$A_{1,2} = \frac{\langle \sigma_{-}(t) \rangle}{2(\lambda_1 - \lambda_2)} = \frac{\langle \sigma_{-}(0) \rangle \sum_i A_i e^{\lambda_i t}}{2(\lambda_1 - \lambda_2)} \{ \cos \phi [\lambda_1 - \lambda_2 \pm (\Gamma_{12} + \Gamma_{21})] \mp$$

$$p_z \cos(\theta + \bar{\theta}) [-2i\delta - \Gamma_2 + \Gamma_1 + \Gamma_{12} - \Gamma_{21}] \}, \quad (6)$$

$$A_{3,4} = \frac{\sin \phi}{2(\lambda_3 - \lambda_4)} \{ \sin \phi [\lambda_3 - \lambda_4 \mp (\Gamma_{34} + \Gamma_{43})] \pm p_z \sin(\theta + \bar{\theta}) [-2i\Omega - \Gamma_4 + \Gamma_3 - \Gamma_{34} + \Gamma_{43}] \}.$$

Eqs. (4)-(6) are the main result of this Communication. They cover the parameters regime where $S(\Omega_{\pm}) \ll \Omega_{\pm}$. Single-phonon processes dominate at low T , whereas multiphonon-exchanges are paramount at higher T where the white noise results of [15] are recovered. Reliability of ME is confirmed by a real-time path-integral calculation.

Discussion of the results.— We focus our analysis on the low-temperature regime $T \ll \Omega_{-}$. Here effects of the dissipative processes internal to the SBE on the qubit behavior are clearly identifiable. In this limit energy absorption processes are exponentially suppressed ($\Gamma_{-}(\Omega_{\pm}) \approx 0$) and the eigenvalues take the forms

$$\begin{aligned} \lambda_1 &= i\delta - \eta_s S(0), \\ \lambda_2 &= -i\delta - \frac{\gamma_{r+} + \gamma_{r+}^0 + \gamma_{r-} + \gamma_{r-}^0}{4} - \eta_s S(0), \\ \lambda_{3,4} &= \pm i\Omega - \frac{\gamma_{r\mp} + \gamma_{r\mp}^0}{4} - \eta_c S(0), \end{aligned} \quad (7)$$

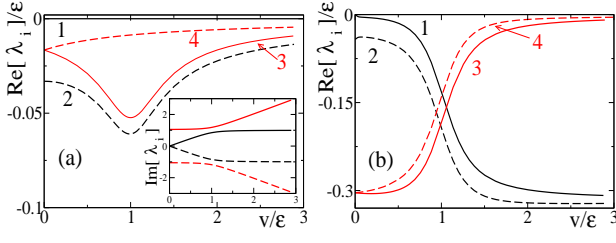


FIG. 2: The four rates $\mathcal{Re}[\lambda_i]$ from Eqs.(4). In (a) $T = 0$. Inset: imaginary parts (independent on temperature for $T < \Omega_+$). In (b) $T = 0.5\Delta$. Parameters are $K = 0.1$, $\varepsilon = 3\Delta$.

where intra-doublet relaxation rates (see Fig. 1)

$$\gamma_{r\pm} = \frac{1}{2} \sin^2(\theta_{\pm}) S(\Omega_{\pm}) = \frac{1}{2} \left(\frac{\Delta}{\Omega_{\pm}} \right)^2 S(\Omega_{\pm}) \quad (8)$$

have been introduced ($\gamma_{r\pm}^0$ value at $T = 0$). Note that pure dephasing processes $\propto S(0)$, are not simple sum of intra-doublet dephasing terms, $\gamma_{\phi\pm} = \frac{1}{2} \cos^2(\theta_{\pm}) S(0)$.

In the following we present a selection of illustrative behaviors for $\varepsilon > \Delta$. In this regime the two conditional Hamiltonians \mathcal{H}_{\pm} may differ significantly and enforce peculiar impurity dynamical behaviors. For example beatings when δ approaches Ω , i.e. around $\varepsilon = v$ which identifies a sort of “resonance regime” for our problem.

We first characterize the asymptotic qubit dynamics, by the T and v dependence of the eigenvalues. At zero temperature the pure dephasing contributions fade away, and one rate, $\mathcal{Re}[\lambda_1]$, vanishes, as expected. Only emission processes contribute to the residual rates, and they directly sound out intra-doublet relaxation rates $\gamma_{r\pm}^0$. Their behaviors reflect the sensitivity of \mathcal{H}_{\pm} , to noise acting along τ_z . While γ_{r+}^0 decreases with increasing v , γ_{r-}^0 takes a maximum at the resonance point (see Eq.(8)), the “transverse” ($\theta_- = \pi/2$) noise condition for \mathcal{H}_- . This implies a non-monotonous dependence of $\mathcal{Re}[\lambda_{2,3}]$ on the coupling v , Fig. 2(a). The imaginary parts of $\lambda_{1,2}$ and $\lambda_{3,4}$ interchange characters at resonance (Fig. 2(a) inset) leading to possible hybridization (see below). Increasing T leading correction to the rates come from pure dephasing terms $S(0)$. As a difference with $T = 0$, all the rates are finite and cross around resonance, Fig. 2(b).

These features are crucial for the asymptotic dynamics of $\langle \sigma_-(t) \rangle$, which does not depend on the impurity preparation. We then expect at $T = 0$, undamped oscillations with δ , while at finite T , damped oscillations driven by one or two complex eigenvalues. For example in the case of Fig. 2(b) the dominant rate is $\mathcal{Re}[\lambda_1]$ for $v < \varepsilon$ and $\mathcal{Re}[\lambda_4]$ for $v > \varepsilon$. It is a non-monotonous function of v and a cusp signals crossing of eigenvalues (a similar effect may explain non-monotonic behavior of [17]).

At intermediate times, all eigenvalues may be relevant, depending on the weights A_i in Eq.(6). To substantiate this point, in Fig. 3 we show A_i corresponding to the eigenvalues in Fig. 2(a) for different preparations. Re-

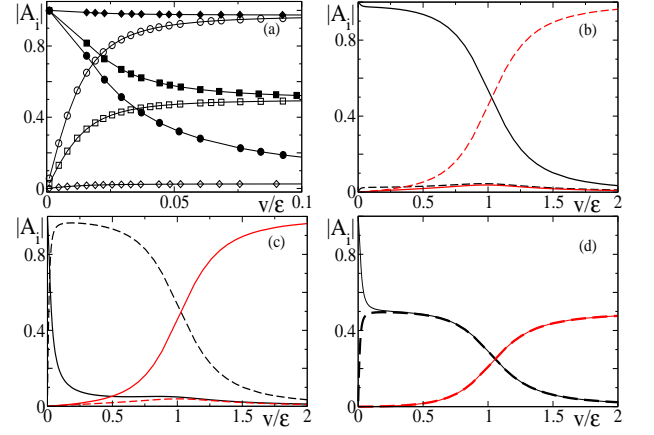


FIG. 3: Weights $|A_i|$ of $e^{\lambda_i t}$ from Eq.(6) as a function of v/ε . (a) Dominant weights in the small v region: $|A_1|$ (full symbols) and $|A_2|$ (open symbols) for $p_z = 0$ (circles), $p_z = -1$ (squares) and $p_z = 1$ (diamonds). Effect of impurity preparations: $p_z = 1$ (b), $p_z = -1$ (c), and $p_z = 0$ (unpolarized state) (d). $|A_1|$ (blue), $|A_2|$ (blue dashed), $|A_3|$ (red), $|A_4|$ (red dashed). Parameters: $T = 0$, $\varepsilon = 3\Delta$, $K = 0.1$.

markably, the weights very weakly depend on T (not shown), then the following picture generally holds for $T < \Omega_-$. For extreme weak coupling, $v \rightarrow 0$, $|A_1| \approx 1$ (Fig. 3(a)) implying universal dynamics independent on the initial conditions. The dominant eigenvalue is λ_1 with $\delta \rightarrow 0$ and $\langle \sigma_-(t) \rangle$ decays exponentially with the Golden rule rate $\Gamma_{GR} = \frac{v^2}{2} \frac{S(0)}{\varepsilon^2 + \Delta^2} \sin^4 \theta$. In this regime the impurity acts as a Gaussian reservoir and may be described with linear response theory in the coupling v . Away from this tiny region non-Gaussian effects occur and different impurity preparations result in different time behaviors, giving separate information on the various eigenvalues. Far from resonance, a single frequency shows up in $\langle \sigma_-(t) \rangle$ independently on p_z (δ if $v < \varepsilon$, Ω if $v > \varepsilon$). Damping of the oscillations depends on the initial condition, Fig. 3 (b) - (d). For instance, at finite $v < \varepsilon$, the decay occurs with $\mathcal{Re}[\lambda_1]$ if $p_z = 1$ and with $\mathcal{Re}[\lambda_2]$ if $p_z = -1$, both rates are present for unpolarized initial state. This behavior is stable against temperature variations. Beatings *and* T -dependence are instead characteristic of the resonant regime. At $v = \varepsilon$, at least two amplitudes are equal, $|A_1| \approx |A_4|$ ($p_z = 1$) or $|A_2| \approx |A_3|$ ($p_z = -1$). Damped beatings at $\Omega_{\pm} = \Omega \pm \delta$ are possible due to the hybridization of $\Omega \approx \delta$ (Fig. 2(a) inset).

We illustrate these features in Fig. 4 for $v = \varepsilon$. The beatings visibility is reduced with increasing T , due the onset of the pure dephasing processes. For an unpolarized state, $p_z = 0$, $\langle \sigma_-(t) \rangle$ shows a intermediate behavior between the ones at $p_z = \pm 1$ since at resonance all eigenvalues contribute (Fig. 3(d)). Damping is strongest for $p_z = -1$, weakest for $p_z = 1$ and intermediate for $p_z = 0$. In fact, for $\varepsilon > \Delta$, preparation in the pure state $p_z = +1$ makes the impurity close to its ground state and

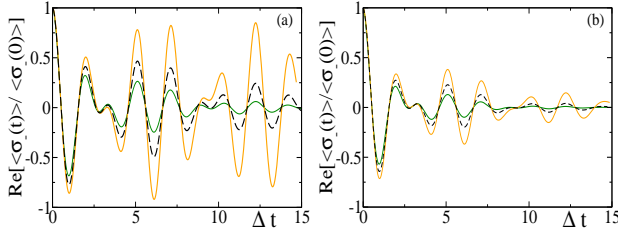


FIG. 4: $\text{Re}[\langle\sigma_{-}(t)\rangle/\langle\sigma_{-}(0)\rangle]$ at resonance $\varepsilon = v = 3\Delta$ for initial states (a) $p_z = 1$: slow decay with $\text{Re}[\lambda_{1/4}]$, (b) $p_z = -1$: fast decay with $\text{Re}[\lambda_{2/3}]$. Parameters: $T = 0$ (blue), $T = 0.5\Delta$ (red), $T = 0.9\Delta$ (green), $K = 0.1$.

less damped, while it is close to the excited state when $p_z = -1$ with strongest damping.

In the last part of this Communication we present an alternative perspective, considering the qubit as a measuring device for a mesoscopic system described by the SBE. Remarkably, the qubit acts as a detector despite the absence of direct qubit-SBE inelastic transitions [19]. In fact, the pure dephasing coupling amounts to a “dispersive”, quantum non-demolition measurement regime for the qubit. Detection is feasible due to the qubit back-action on the SBE. This point of view is illustrated in Fig. 5, where the length of the Bloch vector in the $\hat{x} - \hat{y}$ plane, $|\langle\sigma_{-}(t)\rangle|$, acts as a sensitive detector of the mesoscopic system (“impurity”) preparation. At resonance, the unpolarized state, $p_z = 0$, is identified by beatings, Fig. 5(a). These almost disappear for pure states, $p_z = \pm 1$, where oscillations at Ω_{+} occur, Fig. 5(b). Identification of the impurity preparation far from resonance results instead from different oscillation amplitudes and/or decay rates, Fig. 5 (c) - (d).

In conclusion, we have identified in time domain

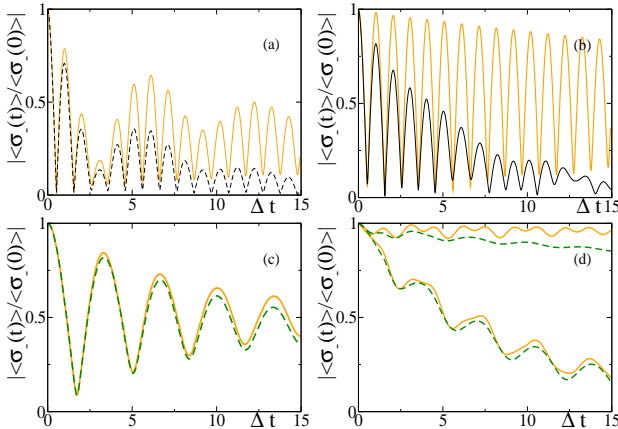


FIG. 5: $|\langle\sigma_{-}(t)\rangle/\langle\sigma_{-}(0)\rangle|$ for $\varepsilon = 3\Delta$, $K = 0.1$. Panels (a) and (b): resonant impurity $v = \varepsilon$. (a) $p_z = 0$ at $T = 0$ (black) and $T = 0.5\Delta$ (red); (b) $T = 0$ for $p_z = 1$ (orange), $p_z = -1$ (black). Panels (c) and (d): non resonant case $v = \Delta$ at $T = 0$ (blue) and at $T = 0.9\Delta$ (red). In (c) $p_z = 0$, in (d) $p_z = 1$ top, $p_z = -1$ bottom. Note the weak T -dependence.

non Gaussian and back-action effects due to a coherent bistable impurity. These may represent a ultimate limitation for solid state qubits even when single shot measurement schemes are available. Our analysis by changing temperature, strain ε and coupling v , may provide valuable insights to realistic scenarios where a wide distribution of the parameters has to be considered [10]. The employed SBE represents a general effective model for complex physical baths awaiting specific microscopic description, as those typical of solid state nanodevices.

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